# REPORT 1128

# CALCULATIONS ON THE FORCES AND MOMENTS FOR AN OSCILLATING WING-AILERON COMBINATION IN TWO-DIMENSIONAL POTENTIAL FLOW AT SONIC SPEED <sup>1</sup>

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#### SUMMARY

The linearized theory for compressible unsteady flow is used, as suggested in recent contributions to the subject, to obtain the velocity potential and the lift and moment for a thin, harmonically oscillating, two-dimensional wing-aileron combination moving at sonic speed. The velocity potential is derived by considering the sonic case as the limit of the linearized supersonic theory. From the velocity potential explicit expressions for the lift and moment are developed for vertical translation and pitching of the wing and rotation of the aileron. The report provides extensive tables of numerical values for the coefficients contained in the expressions for lift and moment, for various values of the reduced frequency k (0< $k \le 3.5$ ), and aileron hinge position (from 10 to 90 percent of the wing chord). sonic results are compared and found to be consistent with previously obtained subsonic and supersonic results. Several figures are presented showing the variation of lift and moment with reduced frequency and Mach number and the influence of Mach number on some cases of bending-torsion flutter.

## INTRODUCTION

Instability investigations for high-speed aircraft often require a knowledge of the air forces and moments that act on an oscillating wing moving at high speed. For subsonic and supersonic speeds the main source of theoretical information has been the solution of the linearized differential equation for compressible flow. For sonic or near-sonic speed, however, the linearized theory has been generally assumed inapplicable, since it does not allow for thickness effects, shocks, and strong disturbances. As is well known, it predicts infinite forces on a nonoscillating, thin, unswept wing moving at sonic speed.

Important differences exist, however, between the steady and unsteady cases. By a discussion of the order of magnitude of the terms of the general nonlinear differential equation for compressible flow, reference 1 shows that for unsteady two-dimensional flow at sonic speed this equation is essentially linear and in linear form leads to physically plausible results for the forces on a thin oscillating wing, provided the

frequency of oscillation is sufficiently large. A similar conclusion was reached in reference 2, where linear methods applied to a wing in two-dimensional nonstationary flow at sonic speed yielded perturbation velocities of the same order of magnitude as those obtained for subsonic or supersonic speeds. In references 3 and 4 expressions and some numerical values are given for the lift forces and moments on an oscillating two-dimensional wing moving at sonic speed. Because of the importance of the sonic problem in present-day flight considerations and because of the insight into the three-dimensional problem that the solution for two-dimensional flow will probably afford, the purpose of the present report is to develop the two-dimensional case more fully.

Consideration is thus given to the case of an oscillating wing-aileron combination in two-dimensional flow at sonic, speed. The velocity potential for this case is obtained, and from the velocity potential expressions for the air forces and moments on the wing-aileron combination are developed in terms of the frequency of oscillation. Numerical tables of the coefficients contained in the expressions for lift and moment are supplied which may be used for the theoretical calculations involved in wing flutter and other instability problems for sonic speed. The tables provide a means for obtaining continuity of calculation between high-subsonic and low-supersonic results for the oscillating wing-aileron combination in two-dimensional flow.

Because of the small-disturbance assumption, the theory and subsequent results are subject to the same restrictions imposed on all small-perturbation theory, subsonic and supersonic. In addition, as the frequency approaches zero, the difficulties of the steady linearized problem are encountered; therefore the validity of the subsequent results is subject to question for the range of low frequencies. Moreover, uncertainty exists because the linear unsteady results are considered to represent disturbances from an equilibrium position that is governed by nonlinear relations, and a great amount of experimentation may be necessary to determine the region of validity for the calculations. Nevertheless, the results serve as a bridge between subsonic and supersonic theory and may be applicable for a range of high frequencies.

<sup>1</sup> Supersedes NACA TN 2590, "Calculations on the Forces and Moments for an Oscillating Wing-Aileron Combination in Two-Dimensional Potential Flow at Sonic Speed" by Herbert C. Nelson and Julian H. Berman, 1952.

#### SYMBOLS

$\boldsymbol{a}$	velocity of sound in undisturbed medium
$\boldsymbol{b}$	wing semichord
$c_1$	section lift coefficient
C <sub>m</sub>	section moment coefficient about leading edge
$f(r_j)$	Fresnel integrals contained in equation (23)
h Tax	vertical displacement of axis of rotation
$J_0(\lambda)$	Bessel function of zero order (first kind)
k	reduced frequency, $\omega b/V$
$k' = \frac{\omega b}{a}$	
$L_i, M_i, N_i$	quantities defined by equation (22); $i=1, 2, 3$ ,
	4, 5, and 6
$L_i', M_i', N_i'$	quantities defined by equation (23), independ- ent of wing-axis-of-rotation position
m	mass of wing per unit span
M	Mach number, $V/a$
$M_{\alpha}$	aerodynamic section moment on wing about
	axis of rotation, positive leading edge up
$M_{eta}$	aerodynamic section moment on aileron about
	its hinge, positive leading edge up
$\stackrel{\Delta p}{P}$	pressure difference
P	aerodynamic section normal force, positive
	downward
$egin{array}{c} t \ V \end{array}$	time
$w^{(2bx,t)}$	flight speed normal velocity at $x$ at time $t$
x, z	nondimensional rectangular coordinates, re-
۷, ۵	ferred to 2b
x'=2bx	101104 00 20
y'=2by	•
z'=2bz	
$x_0$	abscissa of axis of rotation of wing section,
	referred to 2b
$x_1$	abscissa of aileron hinge, referred to 2b
$x_{\alpha}$	location of center of gravity of wing-aileron system measured from elastic axis (see ref. 5)
α	angular displacement (pitch) about axis of rotation
$\alpha_h$	effective angle of attack due to vertical translation, $\hbar/V$
β	angular displacement of aileron, measured relative to $\alpha$
$\theta_h$	phase angle between lift due to $h$ and bending velocity $h$
$\theta_{\sigma}$	phase angle between lift due to $\alpha$ and position $\alpha$
$\theta_{hm}$ .	phase angle between moment due to $h$ and bending velocity $h$
	phase angle between moment due to $\alpha$ and position $\alpha$
κ	density parameter; $\frac{\pi \rho b^2}{m}$ ; reference 5 uses $\mu = \frac{\pi}{4} \frac{1}{\kappa}$
$\xi$ $\xi'=2b\xi$	abscissa of point of disturbance, referred to $2b$

ρ	density in main stream
τ	time variable
$ au_1, \  au_2$	times required for transmittal of disturbance
	to field point
φ	disturbance velocity potential
ω	angular frequency of oscillation
$\omega_h$	natural bending frequency of wing
$\omega_{\alpha}$	natural torsional frequency of wing

#### **ANALYSIS**

The theory presented herein for two-dimensional flow at sonic speed is based on the assumptions that the two wing surfaces act independently and that wake effects are absent. Thus the sonic case as treated is more akin to the supersonic than the subsonic case. The velocity potential for the oscillating two-dimensional wing moving at sonic speed is derived by allowing the Mach number M to approach unity in the velocity potential for the wing moving at supersonic speed. An alternative derivation is also given in which the potential is obtained directly from the linearized differential equation by a method of solution employing the Laplace transformation. In reference 3 Rott obtained the velocity potential by superposition of the elementary source solution of the linearized differential equation.

Velocity potential for sonic speed.—Consider first the velocity potential for a harmonically oscillating two-dimensional wing moving at supersonic speed, given in reference 5 as

$$\phi(2bx,t) = -\frac{2b}{\pi\sqrt{M^2 - 1}} \int_0^x \int_{\tau_1}^{\tau_2} \frac{w(2b\xi,t)e^{-t\omega\tau}d\tau}{\sqrt{(\tau - \tau_1)(\tau_2 - \tau)}}$$
(1)

where

$$\tau_1 = \frac{2b(x-\xi)}{a(M+1)}$$

$$\tau_2 = \frac{2b(x-\xi)}{a(M-1)}$$

a is the speed of sound in the undisturbed medium, x and  $\xi$  are nondimensional coordinates referred to the wing chord 2b,  $w(2b\xi,t)$  is the prescribed local normal velocity at the wing surface, and  $\omega$  is the frequency of oscillation. The integral in equation (1) represents the total effect of all the disturbances created by the wing. The time-lag functions  $\tau_1$  and  $\tau_2$  are associated with the two pulses that occur at the point x because of a disturbance created at the point  $\xi$  (see ref. 5 for more complete discussion). Another form for equation (1), also given in reference 5, is

$$\phi(2bx,t) = -\frac{2b}{\sqrt{M^2 - 1}} \int_0^x w(2b\xi,t) e^{-ix\xi^2 \frac{M(x-\xi)}{M^2 - 1}} J_0\left(2k'\frac{x - \xi}{M^2 - 1}\right) d\xi$$
where  $k' = b\omega$  (2)

As the Mach number M approaches unity, the argument of the Bessel function  $J_0$  in equation (2) becomes infinite and the following asymptotic approximation is applicable:

$$\lim_{M\to 1} J_0\left(2k'\frac{x-\xi}{M^2-1}\right) = \sqrt{\frac{M^2-1}{\pi k (x-\xi)}} \cos\left(2k\frac{x-\xi}{M^2-1} - \frac{\pi}{4}\right). (3)$$

where on the right-hand side k' has been replaced by k since  $k = \frac{b\omega}{V}$  and  $k' = \lim_{M \to 1} k$ . At M = 1 the time-lag function  $\tau_2$  contained in equation (1) becomes infinite and the influence of one of the two pulses characteristic of supersonic flow becomes vanishingly small. (By considering the sonic case as a limit from the subsonic side, the wing at sonic speed cannot overtake the second pulse.)

By letting M approach unity in equation (2) and using equation (3) in the process, the sonic velocity potential is found to be

$$\phi(2bx,t) = -2b \int_0^x w(2b\xi,t) G(x-\xi) d\xi$$
 (4)

where

$$G(x) = \frac{1}{2} \frac{e^{-ikx}}{\sqrt{i\pi kx}}$$

As a matter of possible interest an alternative derivation of equation (4) that makes use of the Laplace transformation (as was done by Stewartson in ref. 6 for supersonic flow) is presented. The linearized differential equation for two-dimensional compressible flow may be written as

$$\frac{1}{a^2} \left( \frac{\partial}{\partial t} + V \frac{\partial}{\partial x'} \right)^2 \phi = \frac{\partial^2 \phi}{\partial x'^2} + \frac{\partial^2 \phi}{\partial z'^2} \tag{5}$$

For the harmonically oscillating wing moving at sonic speed, equation (5) becomes

$$\frac{\partial^2 \psi}{\partial z'^2} = -\frac{\omega^2}{a^2} \psi + \frac{2i\omega}{a} \frac{\partial \psi}{\partial x'} \tag{6}$$

where the disturbance velocity potential  $\phi$  is related to  $\psi$  by

$$\phi(x',z',t) = \psi(x',z') e^{i\omega t}$$

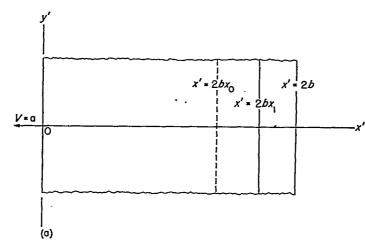
and x'=2bx and z'=2bz. The mean position of the wing (given by z'=0 and  $x'\geq 0$ ) and the rectangular coordinate system being used are moving at velocity V=a in the negative x'-direction, as shown in figure 1. Since this report is concerned only with the lift of a thin wing, the boundary conditions that equation (6) must satisfy are

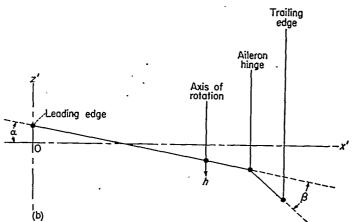
$$\left(\frac{\partial \psi}{\partial z'}\right)_{z'=\pm 0} = w(x') \qquad (x' \ge 0) \quad (7)$$

$$\psi \rightarrow 0 \text{ as } z' \rightarrow \pm \infty$$
 . (8)

$$\psi = 0 \qquad (x' < 0) \quad (9)$$

In accordance with small-disturbance linearized theory the boundary conditions are expressed for the mean position of





(a) Projection of wing strip on x'y'-plane.

(b) Section y'=0.

FIGURE 1.—Sketch illustrating coordinate system and the degrees of freedom  $\alpha$ , h, and  $\beta$ .

the wing rather than the wing itself. Equation (7) implies that the normal-velocity distribution on the wing is given; equation (8) is a condition on the behavior at infinity (the manner of approaching zero is associated with the radiation condition of Sommerfeld); equation (9) is the condition that no disturbances be propagated forward of the wing. Since the velocity potential must be continuous, equation (9) implies that

$$\psi(+0,z') = \psi(-0,z') = 0$$

Equations (6) to (9) constitute the boundary-value problem for the velocity potential  $\phi$ .

Applying the Laplace transform

$$\overline{\psi}(s,z') = \int_0^\infty e^{-sx'} \psi(x',z') dx'$$

to equations (6) to (9) yields

$$\left(\frac{d^2\overline{\psi}}{dz'^2}\right) = \left(\frac{2i\omega}{a}, s - \frac{\omega^2}{a^2}\right) \overline{\psi} = \mu^2 \overline{\psi}$$
 (10)

$$\left(\frac{d\overline{\psi}}{dz'}\right)_{z'=\pm 0} = w(s) \tag{11}$$

$$\overline{\psi} \rightarrow 0 \text{ as } z' \rightarrow \pm \infty$$
 (12)

From equations (10) to (12) the value for  $\overline{\psi}$  is

$$\overline{\psi} = -\frac{z'}{|z'|} \frac{w(s)e^{-\mu x'}}{\mu} \tag{13}$$

From equation (13) the value for  $\overline{\psi}$  at the upper surface of the wing (z'=+0) is

$$\overline{\psi} = -\frac{w(s)}{\mu} \tag{14}$$

Applying the inverse transform to equation (14) yields

$$\psi(x',+0) = -\int_0^{x'} w(\xi')G(x'-\xi')d\xi'$$

or

$$\phi(x', +0, t) = -e^{t\omega t} \int_0^{x'} w(\xi') G(x' - \xi') d\xi'$$
 (15)

where

$$G(x') = \frac{1}{2} \frac{e^{-\frac{i\omega}{2a}x'}}{\sqrt{\pi \frac{i\omega}{2a}x'}}$$

$$E' = 2b\xi$$

Equations (15) and (4) are identical, each giving the velocity potential at the upper surface of the wing.

Application to wing-aileron combination.—For the particular case of the wing-aileron combination oscillating harmonically in vertical translation h, pitch  $\alpha$ , and aileron rotation  $\beta$  (see fig. 1(b)), the normal velocity at a point x of the wing chord may be expressed as

$$w(2bx,b) = -[\hbar + V\alpha + 2b(x-x_0)\dot{\alpha} + V\beta + 2b(x-x_1)\dot{\beta}]$$
 (16)

where

$$h = h_0 e^{i\omega t}$$

$$\alpha = \alpha_0 e^{i\omega t}$$

$$\beta = \beta_0 e^{i\omega t}$$

 $h_0$ ,  $\alpha_0$ , and  $\beta_0$  are complex amplitudes, and the  $\beta$  terms are to be interpreted as zero for  $x < x_1$ . Since linearized theory is being employed, the potential given in equation (4)

may be considered as the sum of five potentials, each of which is associated with one of the terms of the right-hand side of equation (16). Hence the potential may be written as

$$\phi = \phi_{\dot{h}} + \phi_{\alpha} + \phi_{\dot{\alpha}} + \phi_{\beta} + \phi_{\dot{\beta}} \tag{17}$$

where upon substituting equation (16) into equation (4)

$$\phi_{\dot{\alpha}} = 2bh \int_0^x G(x-\xi)d\xi$$

$$\phi_{\alpha} = 2bV\alpha \int_0^x G(x-\xi)d\xi$$

$$\phi_{\dot{\alpha}} = 4b^2\dot{\alpha} \int_0^x (\xi-x_0)G(x-\xi)d\xi$$

$$\phi_{\dot{\beta}} = 2bV\beta \int_{x_1}^x G(x-\xi)d\xi$$

$$\phi_{\dot{\beta}} = 4b^2\beta \int_{x_1}^x (\xi-x_1)G(x-\xi)d\xi$$

Forces and moments.—The velocity potential for the upper wing surface given in equation (4) is antisymmetric with respect to the plane z=0, as may be noted in the boundary condition (eq. (7)). The local pressure difference, positive downward, between the upper and lower surfaces of the wing at any point x is obtained from equation (4) by means of

$$\Delta p = -2\rho \left( \frac{\partial \phi}{\partial t} + \frac{V}{2b} \frac{\partial \phi}{\partial x} \right)$$

where  $\rho$  is the density of the undisturbed medium. The force (positive downward) acting on a wing section is therefore

$$P = 2b \int_0^1 \Delta p \ dx \tag{18}$$

The moments (positive leading edge up) acting on the entire wing section about the axis of rotation at  $x_0$  and on the aileron section about the hinge point  $x_1$  are, respectively,

$$M_{\alpha} = 4 b^{2} \int_{0}^{1} (x - x_{0}) \Delta p \ dx \tag{19}$$

$$M_{\beta} = 4b^2 \int_{x_1}^{1} (x - x_1) \Delta p \ dx$$
 (20)

Upon substituting equation (17) into equations (18), (19), and (20) and performing the indicated integrations, the results may be written as

$$P = -4 \rho b V^{2} k^{2} e^{i\omega t} \left[ \frac{h_{0}}{b} (L_{1} + iL_{2}) + \alpha_{0}(L_{3} + iL_{4}) + \beta_{0}(L_{5} + iL_{6}) \right]$$

$$M_{\alpha} = -4 \rho b^{2} V^{2} k^{2} e^{i\omega t} \left[ \frac{h_{0}}{b} (M_{1} + iM_{2}) + \alpha_{0}(M_{3} + iM_{4}) + \beta_{0}(M_{5} + iM_{6}) \right]$$

$$M_{\beta} = -4 \rho b^{2} V^{2} k^{2} e^{i\omega t} \left[ \frac{h_{0}}{b} (N_{1} + iN_{2}) + \alpha_{0}(N_{3} + iN_{4}) + \beta_{0}(N_{5} + iN_{6}) \right]$$
(21)

The coefficients of equations (21) can be expressed as follows with primed quantities introduced for convenience in numerical tabulation to denote terms independent of the wing-axis-of-rotation position  $x_0$  (referred to  $x_0=0$ ):

$$L_{1}+iL_{2}=L_{1}'+iL_{2}'$$

$$L_{3}+iL_{4}=L_{3}'+iL_{4}'-2x_{0}(L_{1}'+iL_{2}')$$

$$L_{5}+iL_{6}=L_{5}'+iL_{6}'$$

$$M_{1}+iM_{2}=M_{1}'+iM_{2}'-2x_{0}(L_{1}'+iL_{2}')$$

$$M_{3}+iM_{4}=M_{3}'+iM_{4}'-2x_{0}[(M_{1}'+iM_{2}')+(L_{3}'+iL_{4}')]+4x_{0}^{2}(L_{1}'+iL_{2}')$$

$$M_{5}+iM_{6}=N_{5}'+iN_{6}'+2(x_{1}-x_{0})(L_{5}'+iL_{6}')$$

$$N_{1}+iN_{2}=N_{1}'+iN_{2}'+M_{1}'+iM_{2}'-2x_{1}(L_{1}'+iL_{2}')$$

$$N_{3}+iN_{4}=N_{3}'+iN_{4}'-2x_{0}(N_{1}+iN_{2})$$

$$N_{5}+iN_{6}=N_{5}'+iN_{6}'$$

$$(22)$$

The primed quantities, as a result of integration by parts, can be expressed as

$$L_{1}'+iL_{2}'=-\frac{1-i}{r_{0}}f(r_{0})+\frac{1+i}{r_{0}^{2}}\sqrt{\frac{r_{0}}{2\pi}}e^{-ir_{0}}$$

$$L_{5}'+iL_{4}'=\frac{1-i}{2r_{0}}\left(-2+\frac{2i}{r_{0}}+\frac{1}{2r_{0}^{2}}\right)f(r_{0})+\frac{1+i}{2r_{0}^{2}}\sqrt{\frac{r_{0}}{2\pi}}e^{-ir_{0}}\left(2-\frac{i}{r_{0}}\right)$$

$$L_{6}'+iL_{6}'=(1-x_{1})^{3}\left[\frac{1-i}{2r_{1}}\left(-2+\frac{2i}{r_{1}}+\frac{1}{2r_{1}^{2}}\right)f(r_{1})+\frac{1+i}{2r_{1}^{2}}\sqrt{\frac{r_{1}}{2\pi}}e^{-ir_{1}}\left(2-\frac{i}{r_{1}}\right)\right]$$

$$M_{1}'+iM_{2}'=\frac{1-i}{2r_{0}}\left(-2-\frac{1}{2r_{0}^{2}}\right)f(r_{0})+\frac{1+i}{2r_{0}^{2}}\sqrt{\frac{r_{0}}{2\pi}}e^{-ir_{0}}\left(2-\frac{i}{r_{0}}\right)$$

$$M_{3}'+iM_{4}'=\frac{1-i}{2r_{0}}\left(-\frac{8}{3}+\frac{2i}{r_{0}}-\frac{i}{2r_{0}^{3}}\right)f(r_{0})+\frac{1+i}{2r_{0}^{2}}\sqrt{\frac{r_{0}}{2\pi}}e^{-ir_{0}}\left(\frac{8}{3}-\frac{2i}{3r_{0}}+\frac{1}{r_{0}^{2}}\right)$$

$$N_{1}'+iN_{2}'=x_{1}^{3}\left[\frac{1-i}{2r_{2}}\left(-2+\frac{1}{2r_{2}^{2}}\right)f(r_{2})+\frac{1+i}{2r_{2}^{2}}\sqrt{\frac{r_{2}}{2\pi}}e^{-ir_{2}}\left(2+\frac{i}{r_{2}}\right)\right]$$

$$N_{3}'+iN_{4}'=x_{1}^{4}\left[\frac{1-i}{2r_{2}}\left(-\frac{4}{3}+\frac{2i}{r_{2}}+\frac{1}{r_{2}^{2}}+\frac{i}{2r_{2}^{2}}\right)f(r_{2})+\frac{1+i}{2r_{2}^{2}}\sqrt{\frac{r_{2}}{2\pi}}\left(\frac{4}{3}-\frac{4i}{3r_{2}}-\frac{1}{r_{2}^{2}}\right)e^{-ir_{2}}\right]+M_{3}'+iM_{4}'-2x_{1}(L_{5}'+iL_{4}')$$

$$N_{5}'+iN_{6}'=(1-x_{1})^{4}\left[\frac{1-i}{2r_{1}}\left(-\frac{8}{3}+\frac{2i}{r_{1}}-\frac{i}{2r_{1}^{3}}\right)f(r_{1})+\frac{1+i}{2r_{1}^{2}}\sqrt{\frac{r_{1}}{2\pi}}\left(\frac{8}{3}-\frac{2i}{3r_{1}}+\frac{1}{r_{1}^{2}}\right)e^{-ir_{1}}\right]$$

where

$$r_0 = k$$

$$r_1 = (1 - x_1) k$$

$$r_2 = x_1 k$$

and the quantities  $f(r_i)$  are the Fresnel integrals

$$f(r_j) = \int_0^{r_j} \frac{e^{-tx}}{\sqrt{2\pi x}} dx \qquad (j = 0, 1, 2)$$

The primed quantities  $L_{i'}$  and  $M_{i'}$  (i=1, 2, 3, and 4), associated with wing bending torsion, are tabulated in table I

as functions of the reduced frequency k for the range  $0 < k \le 3.5$ . The primed quantities  $L_i'$ ,  $M_i'$  (i=5 and 6), and  $N_i'$  (i=1, 2, 3, 4, 5, and 6), introduced by the aileron degree of freedom are tabulated in or can be obtained from table II for the same values of k and for values of the aileron hinge position  $x_1$  ranging from 0.1 to 0.9 in increments of 0.1. In order to make the tabulated values more uniform, each of the primed quantities listed in the tables has been multiplied by the reduced frequency squared  $k^2$ , which appears in the force and moment equations (eqs. (21)).

## DISCUSSION

Lift forces and moments.—The lift forces and moments, the coefficients of which are given in table I, apply to a thin, oscillating, two-dimensional wing moving at sonic speed. A comparison of these results with the forces and moments previously obtained for the same type of wing moving at subsonic and supersonic speeds (refs. 5, 7, 8, and other-papers) may be of interest.

For purposes of comparison, consider the case of a wing pitching about its leading edge and translating vertically. The lift coefficient  $c_l$  and the moment coefficient about the leading edge  $c_m$  can be expressed as

$$c_{l} = -\frac{P}{\rho b V^{2}} = 4 \left[ -ik \left( L_{1}' + iL_{2}' \right) \alpha_{h} + k^{2} \left( L_{3}' + iL_{4}' \right) \alpha \right]$$

$$c_{m} = \frac{M_{\alpha}}{2\rho b^{2} V^{2}} = -2 \left[ -ik \left( M_{1}' + iM_{2}' \right) \alpha_{h} + k^{2} \left( M_{3}' + iM_{4}' \right) \alpha \right]$$
(24)

where  $\alpha_i = \frac{\dot{h}}{V}$  is the angle of attack due to vertical translation and the quantities  $L_{i'}$  and  $M_{i'}$  are now dependent on M as well as k. For the nonoscillating wing in incompressible flow (k=0,M=0)  $c_i=2\pi\alpha$  and  $c_m=-\frac{\pi}{2}\alpha$ . From equation (24) the lift- and moment-curve slopes (complex derivatives) associated with vertical translation and pitching are, respectively,

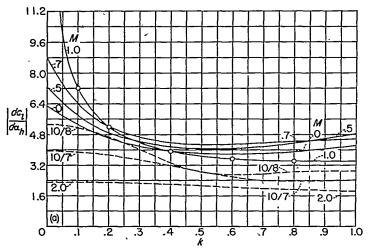
$$\frac{dc_{1}}{d\alpha_{h}} = -i4k (L_{1}' + iL_{2}')$$

$$\frac{dc_{m}}{d\alpha_{h}} = i2k (M_{1}' + iM_{2}')$$

$$\frac{dc_{1}}{d\alpha} = 4k^{2} (L_{3}' + iL_{4}')$$

$$\frac{dc_{m}}{d\alpha} = -2k^{2} (M_{3}' + iM_{4}')$$
(25)

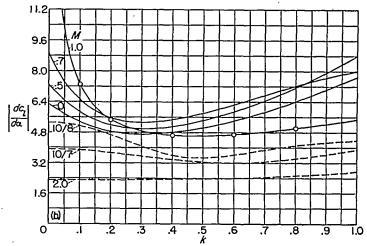
In figure 2 the magnitudes of the slopes given by equation (25) are plotted against k for several values of M, and in figure 3 the associated phase angles are plotted.



(a) Lift-curve slope associated with vertical translation of wing.

$$\left|\frac{dc_l}{d\alpha_h}\right| = 4k\sqrt{L_1'^2 + L_2'^2}.$$

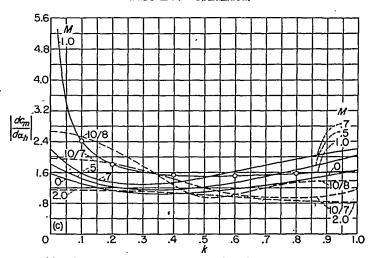
FIGURE 2.—Magnitude of lift-curve slope and moment-curve slope against reduced frequency for several Mach numbers.



(b) Lift-curve slope associated with pitching of wing.

$$\left| \frac{dc_l}{d\alpha} \right| = 4k^2 \sqrt{L_2'^2 + L_4'^2}$$
.

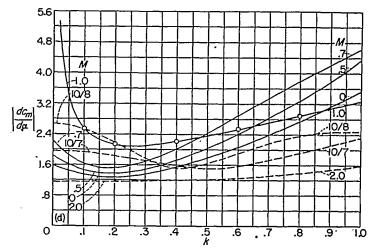
FIGURE 2.—Continued.



(c) Moment-curve slope associated with vertical translation.

$$\left|\frac{dc_m}{d\alpha_k}\right| = 2k\sqrt{M_1'^2 + M_2'^2}.$$

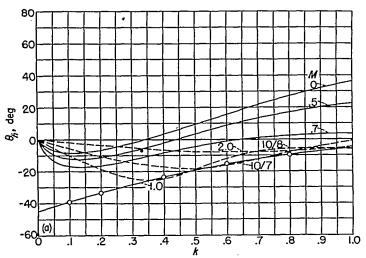
FIGURE 2.—Continued.



(d) Moment-curve slope associated with pitching of wing.

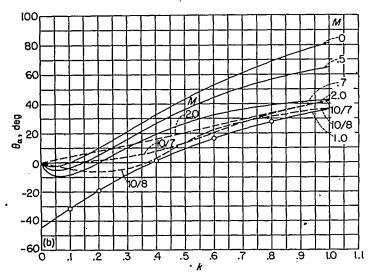
$$\frac{|dc_{+}|}{|d\alpha|} = 2k^2 \sqrt{M_3'^2 + M_4'^2}.$$

FIGURE 2.—Concluded.



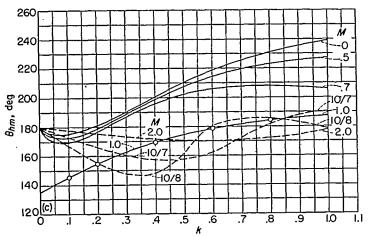
(a) Phase angle between lift vector due to vertical translation and vertical velocity vector  $\hbar$ .

FIGURE 3.—Phase angles plotted against reduced frequency for several Mach numbers.



(b) Phase angle between lift vector due to pitching and angular displacement vector  $\alpha$ .

FIGURE 3.—Continued.



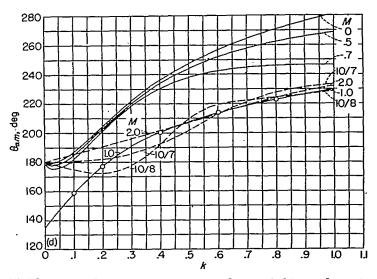
(c) Phase angle between moment vector due to vertical translation and vertical velocity vector  $\hbar$ .

FIGURE 3.—Continued.

In figures 2 and 3 the dashed curves represent the supersonic results, the solid-line curves represent the subsonic results, and the solid-line curves with several of the computed points circled represent the sonic results.

In figure 2 the variation of slope with Mach number for the steady case (along ordinate k=0) is given by the Prandtl-Glauert rule for subsonic speeds and the Ackeret rule for supersonic speeds. Each of these rules predicts an infinite slope at M=1. In the figure, the values for the slope magnitude become excessive only for Mach numbers approaching unity and values of k approaching zero. In this neighborhood the linearized theory does not apply, and the Mach number and k range in which the theory is applicable awaits experimental or theoretical determination. In figure 3 the phase-angle curves for M=1 depart from those for the other Mach numbers in the low k range. At k=0, the phase angle for M=1 differs from the constant phase angle of all the other Mach numbers by 45°.

Figure 4 contains a cross plot against Mach number of figure 2 (a) for several values of k. Note that the maximum lift-curve slope occurs at M=1 only for small values of k.



(d) Phase angle between moment vector due to pitching and angular displacement vector  $\alpha$ .

FIGURE 3.—Concluded.

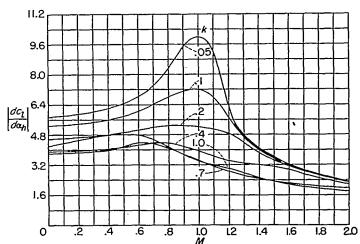


Figure 4.—Magnitude of lift-curve slope associated with vertical translation of wing against Mach number for several values of reduced frequency.  $\left|\frac{dc_1}{dt_1}\right| = 4k\sqrt{L_1'^2 + L_2'^2}$ .

Above a k of around 0.2, as may also be noted in figure 2 (a), the maximum lift-curve slope for a particular value of k occurs at a Mach number less than 1.

Some applications to bending-torsion flutter.—In reference 5 a systematic numerical study of the bending-torsion flutter of a two-dimensional wing was made including, among other considerations, the effect of Mach number on this type of flutter. The results were presented in the form of figures. Table I of the present report is used to obtain points at M=1 for figures 18 and 19 of reference 5. These figures of reference 5 with the M=1 points added are presented as figures 5 and 6.

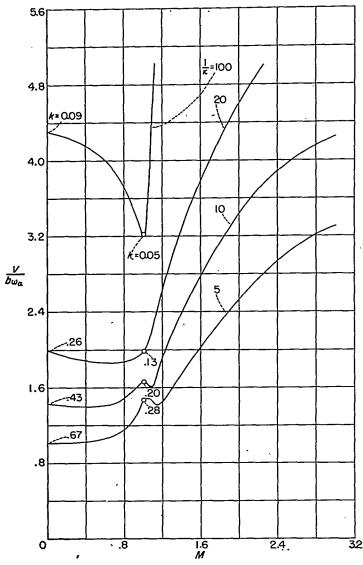


FIGURE 5.—Flutter-speed coefficient against Mach number for several values of  $1/\kappa$  when  $\frac{\omega_h}{\omega_a} = 0$ ,  $x_a = 0.2$ , and a = 0. (Fig. 18 of ref. 5 modified to include calculated values indicated by circles.)

In figure 5 the flutter-speed coefficient  $V/b\omega_a$  is plotted against Mach number M for several values of the density parameter  $1/\kappa$ , for wings with the center of gravity at 60 percent chord and the elastic axis at 50 percent chord. The points for Mach number 1, indicated by circles, are consistent with the results of reference 5. As a matter of possible interest some values of the reduced frequency are indicated at M=0 and M=1.

In figure 6 a plot of the flutter-speed coefficient  $V/b\omega_a$  against the ratio of wing bending frequency to wing torsional frequency  $\omega_h/\omega_a$  is shown for several Mach numbers. The curve for M=1, calculated points of which are circled, is shown in relation to the curves previously given in reference 5.

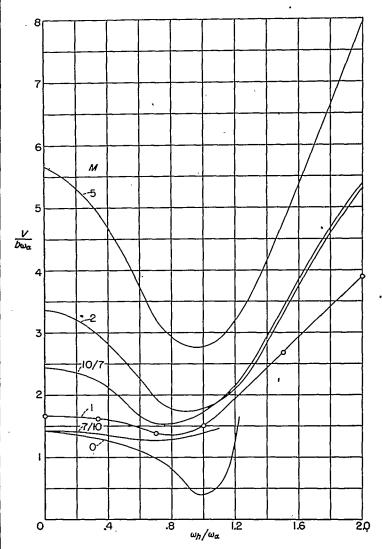


FIGURE 6.—Flutter-speed coefficient against frequency ratio for several values of M when a=0,  $x_a=0.2$ , and  $\frac{1}{\kappa}=10$ . (Fig. 19 of ref. 5 modified to include calculated values indicated by circles.)

## CONCLUDING REMARKS

The linearized theory for compressible unsteady flow has been used to obtain the forces and moments for a thin, harmonically oscillating, two-dimensional wing-aileron combination moving at sonic speed. These forces and moments and the flutter results obtained from them were found to be consistent with similar calculations previously obtained for other Mach numbers. In assessing or applying the results for a Mach number of 1, the limitations associated with linearized theory should be kept in mind. In addition, aspect ratio considerations become increasingly important as the Mach number approaches 1 and may render the two-dimensional results inapplicable to a finite wing even for high frequencies.

Langley Aeronautical Laboratory, National Advisory Committee for Aeronautics, Langley Field, Va., September 4, 1951.

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TARLE	T	SHITLIAY.	OF	THNCTIONS	$\mathbf{F} \cap \mathbf{F}$	WING	RETTTER	CALCULATIONS

k	k2L1'	k4L1'	k³La′	₽L(	$k^{3}M_{1}'$	<i>k</i> 2M2′	k2M1'	<i>k</i> 2M4′
3, 5	-0.092268	3, 5956	0, 97979	3. 4024	-0. 18699	3, 8153	0.92484	4. 6249
3.0	13457	3,0365	. 99531	2,8776	25228	3. 2404	. 93852	3.9240
2.5	14084	2.4591	1. 0199	2.3562	31793	2.6183	. 96756	3. 2302
20	- 10268	1.8845	1.0503	1.8409	-, 31343	1. 9795	1.0071	2.5514
ĩ. š	023461	1.3390	1.0808	1. 3296	- 23499	1.3642	1.0459	1.8920
1. ŏ	.077100	.84912	1. 1048	. 80532	10151	. 81581	1,0653	1. 2439
.8	.11455	. 67354	I. 1184	. 57971	042407	62412	1.0625	. 97907
.6	14412	. 51052	1. 1261	32969	.013043	45119	1.0525	.69934
.4	15861	. 35939	1. 1582	.024216	.057482	29803	1.0398	38091
.36	.15871	.33031	1, 1709	- 019992	.064095	26975	1. 0387	.30774
.32	15754	.30149	L 1875	13156	069652	24221	1.0392	. 22918
.28	15489	. 27281	1. 2101	- 22295	073971	21535	1.0419	14346
.24	15050	. 24413	1. 2414	32793	076821	18912	1.0488	.047825
.20	.14396	. 21524	1. 2862	45272	077892	. 16338	1.0627	- 062204
.18	. 13974	.20060	1. 3163	52578	.077634	. 15065	1.0027	12488
. 16	. 13475	. 18577	1. 3538	60859	.076751	13796	1.0890	- 19460
. 14	12887	.17064	1, 4015	70450	.075154	12527	1.1101	- 27367
.12	12195	18810	1. 4638	81857		. 11249	1. 1399	- 36562
.10	.12195	. 15510 . 13898	1.5480	95913	.072717	. 099537	1. 1830	47627
.09		10000	1.6023	-1. 0437	.067098	. 092939	1. 1830	54159
.08	. 10907 . 10394	. 13061 . 12198	1,6681	-1.0437 -1.1414	.064570	. 086227	1.2482	61613
.07	. 10394	.11302	1. 7495	-1. 2569			1. 2941	70295
.08		.10364	1. 8530	-1.3969	.061627	.079356	1. 3541	80683
	. 091915	.093703	1.9895				1. 4352	93558
05 .045	. 084783	.088464	1.9895 2.0756	-1. 5732 -1. 6808	. 054173 . 051891	. 064875	1. 4873	-1.0133
	.080848				168100		1. 5503	-1.1019
.04	.076618	. 083002	2.1789	-1.8066	. 049394	. 057053	1, 6283	-1. 2103
. 035	. 072039	.077263	2, 3050 2, 4632	-1.9566	. 046644	. 048569	1. 7271	-1.3402
.03	. 067037	.071182	2, 4632	-2.1407 -2.3749	. 043593	. 043954	1. 8573	-1.5041
. 025	. 061508	. 064663			. 040169		1.8885	-1.5424
. 024	.060324	. 063291	2.7184 2.7708	-2,4300 -2,4885	. 039431	. 042991 . 042013	1, 9219	-1.5830
.023	059116		2.7708 2.8269	-2.4886 -2.5508		.042013	1. 9578	-1.6350 -1.6262
. 022	.057877	. 060481	2.8209 2.8871	-2.6008 -2.6173	. 037893	040004	1.9961	-1.6723
. 021	056602	. 059032	2.8871 2.9519	-2.6173 -2.6886	. 037091	038972	2.0376	-1.7216
.020	. 055296			-2. 0886 -2. 7653			2.0825	-1. 7746
.019	. 053948	. 056038	3. 0219		.035411	.037919	2.1314	-1.8316
.018	. 052563	.054490	3.0978	-2.8480	.034532	. 036842	2.1314 2.1847	-1.8316 -1.8934
.017	. 051133	.052901	3, 1804	-2.9377	. 033619	. 035741	2.1847	-1.8834
.016	. 049656	. 051272	3. 2712	-3,0356	.032676	. 034614	2,3078	-1.9006 -2.0341
.015	. 048130	. 049595	3.3710	-3.1428	.031696	. 033455	2,3078	-2.1150
.014	. 046544	. 047865	3. 4814	-3. 2610	. 030678		2,3791	-2.1160 -2.2048
.013	. 044897	. 046080	3.6048	-3.3925	.029617	.031035	2.4695 2.5498	-2.2048 -2.3050
.012	.043178	.044227	3.7434	-3, 5395	. 028506	. 029765		-2.3050 -2.4183
.011	. 041382	. 042303	3,9012	-3, 7059	.027344	.028448	2.6528	-2.4183 -2.5476
. 010	. 039496	. 040294	4, 0823	-3, 8961	.026118	:027076	2, 7710	-2.04/6

REPORT 1128—NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS
TABLE II.—VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS<sup>2</sup>

(wing chords)	k <sup>2</sup> L <sub>1</sub> '	la La'	k2N1	₺³N₂	£2N3'	k²N(′	k²N{′	E2N4			
	k=3.5										
0.1 .2 .3 .4 .5 .6 .7 .8	0.89067 .80334 .71591 .62642 .53285 .43448 .33082 .22375 .11746	2, 7314 2, 1347 1, 6131 1, 1660 . 79307 . 49071 . 25786 . 991788 —. 0069616	-0.093390450420065088 .018221 .028928 .029296 .022080 .012059 .0034955	3. 1344 2. 5061 1. 9349 1. 4282 . 99321 . 63394 . 35459 . 15626 . 038640	0.74199 .68009 .43962 .32026 .22116 .14121 .07954 .035526 .0089560	3.9401 3.2668 2.6203 2.0135 1.4603 .97433 .57050 .26349 .068345	0. 75549 .60700 .47589 .36957 .25674 .10833 .045843 .042332 .010386	3.3484 2.3329 1.5491 .96683 .55513 .22195 .11782 .033681 .0028866			
,		<b>.</b>		k=3.0							
0.1 .3 .4 .5 .6 .7 .8	0. 90819 .82052 .73081 .63775 .54039 .42845 .33269 .22522 .11980	2, 3077 1, 8021 1, 3604 , 98181 , 66478 , 40760 , 20837 , 065938 —, 017583	-0. 20128 14342 062088 051800 023670 0068100 . 00094230 . 0025070 . 0011246	2. 6676 2. 1398 1. 6590 1. 2308 . 86078 . 55321 . 31164 . 13836 . 034469	0. 75299 . 88236 . 44511 . 32322 . 22208 . 14093 . 078792 . 034885 . 0087147	3.3431 2.7714 2.2224 1.7075 1.2382 .82635 .48402 .22367 .058061	0.77297 . 62394 . 48942 . 38851 . 26148 . 16982 . 095332 . 042098 . 010400	2.8401 1.9798 1.3160 .82271 .47300 .24055 .10013 .027973 .0018738			
				k=2.5							
0.1 .2 .3 .4 .5 .6 .7 .8	0, 93119 . 84025 . 74619 . 64844 . 54688 . 44193 . 33476 . 22755 . 12326	1. 8879 1. 4727 1. 1096 . 79775 . 53640 . 32213 . 15607 . 037479 — 030013	-0. 26703 20750 15129 10298 064825 036076 017119 0061219 0011501	2. 1572 1. 7337 1. 3481 1. 0040 . 70575 45578 . 25831 . 11543 . 028958	0. 77781 .60881 .46103 .33476 .22984 .14633 .080881 .035620 .0088363	2,7522 2,2808 1,8281 1,4039 1,0179 67894 .39763 .18376 .047711	0. 70913 .64466 .50353 .37653 .28473 .17045 .095456 .041820 .010466	2. 3397 1. 6329 1. 0874 . 68113 . 39246 . 19903 . 082000 . 021904 . 00072869			
	·			£=2.0	,	1					
0.1 .3 .4 .5 .6 .7 .8	0, 95660 .86004 .76036 .65763 .55240 .44536 .33782 .23164 .12862	1. 4727 1. 1456 . 85876 . 61140 . 40268 . 23188 . 098908 . 0049432 — . 045272	0. 27189 22041 16926 12269 050992 027244 011370 0026384	1. 6293 1. 3098 1. 0197 . 76104 . 53616 . 34768 . 19788 . 088860 . 022415	0. 81216 .63808 .49488 .35314 .24290 .15385 .085640 .037677 .0093252	2. 1748 1. 8017 1. 4432 1. 1075 80216 .53476 .31294 .14464 .037512	0. 82913 .66496 .51552 .38209 .26633 .17000 .094720 .041692 .010627	1. 8511 1. 2943 . 86344 . 51124 . 31088 . 15666 . 062940 . 016236 —. 00062204			
				k=1.5							
0.1 .2 .3 .4 .5 .6 .7 .8	0. 97994 . 87691 . 77191 . 66539 . 55793 . 48043 . 34394 . 23960 . 13761	1. 0577 .81520 .60170 .41675 .26010 .13188 .032740 —.035165 —.065466	-0. 21153 17744 14113 10616 074653 047957 026874 011820 0029068	1. 1188 . 89766 . 69811 . 52088 . 36716 . 23837 . 13591 . 061184 . 015481	0.84629 .66784 .50999 . .37404 .25792 .16412 .091712 .040478 .010045	1. 6153 1. 3386 1. 0719 . 82193 . 59472 . 39602 . 23148 . 10679 . 027684	0. 85440 . 67928 . 52182 . 38333 . 26516 . 16839 . 093816 . 041603 . 010987	1. 3749 .96219 .64145 .40055 .22779 .11189 .042001 .0074945 —.0023279			
				k=1.0							
. Q1 .2 .3 .4 .5 .6 .7 .8	0. 99808 . 89071 . 78309 . 67565 . 56887 . 46329 . 35939 . 25724 . 15480	0. 62511 .46377 .32129 .19781 .003697 .0096882 —.052749 —.090548 —.095913	-0. 10111 090344 075513 05929 043368 028919 016896 0076650 0019552	0. 66383 . 52938 . 41001 . 30482 . 21425 . 13880 . 079021 . 035540 . 0089892	0. 86349 . 68378 . 52446 . 38673 . 26795 . 17142 . 096326 . 042741 . 010663	1. 0685 . 88808 . 71223 . 54649 . 39542 . 26317 . 18370 . 070828 . 018337	0. 86251 . 68000 . 51857 . 37888 . 26141 . 16637 . 093606 . 042507 . 011830	0. 90122 .62662 .41259 .25176 .13690 .060946 .016863 —.0024881 —.0047628			
				<b>k=</b> 0.8			· -				
0.1 .2 .3 .4 .5 .6 .7 .8	1. 0058 .89824 . 79104 . 68454 . 57912 . 47503 . 37242 . 27076 . 16681	0. 43534 .30614 .19227 .094067 .012108 052625 098381 12172 11414	-0.050987 -0.049079 -0.043885 -0.03887 -0.027084 -0.01542 -0.011024 -0.051309 -0.013332	0. 50518 .40143 .30975 .22961 .16099 .10405 .059124 .026545 .0067040	0. 86067 68186 52362 38556 268377 17194 096806 048044 010760	0. 84646 .70630 .56794 .43656 .31627 .21067 .12310 .056737 .014689	0. 85802 . 67520 . 51439 . 37595 . 25995 . 16625 . 094394 . 043561 . 012482	0. 70455 .48465 .31359 .18566 .095226 .036670 .0043043 —.0077843 —.0061613			

<sup>&</sup>lt;sup>2</sup>Corrections to table II of NACA TN 2590 have been incorporated herein.

TABLE II.—VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS2—Continued

(wing chords)	$k^2L_i'$	k2L4'	₽Ni	$k^2N_2$	£³N₃′	₽N/	ĿN.	PN		
k=0.6										
0.1 .2 .3 .4 .5 .6 .7 .8	1. 0191 91274 80716 70250 59900 49655 39488 29274 .18530	0, 22164 .12542 .041375 029996 087916 13117 16773 16371 13969	-0.0032162 010365 012874 012425 010642 0078988 0049936 0024409 00066060	0. 36259 . 28651 · . 22003 . 16257 . 11344 . 073069 . 041388 . 018527 . 0046687	0. 85075 67352 51710 88102 26534 17025 095990 042746 010705	0. 61351 .51660 .41810 .32298 .23487 .15691 .091897 .042430 .011000	0. 84920 . 66834 . 50994 . 37393 . 26002 . 16781 . 096638 . 045598 . 013541	0. 48383 .33006 .20378 .11079 .046840 .00776522 —.011239 —.014625 —.0080683		
k=0.4										
0.1 .2 .3 .4 .5 .6 .7 .8	1. 0538 . 95006 . 84709 . 74483 . 64310 . 54152 . 43912 . 33362 . 21789	-0. 044994 10525 15606 19675 22637 24344 24557 22829 18066	0. 036142: . 022694 . 013670 . 0076867 . 0038858 . 0016488 . 00049747 . 000046360 —. 000029310	0. 23706 . 18582 . 14172 . 10395 . 072186 . 046250 . 026066 . 011615 . 0029130	0. 83622 . 66002 . 50573 . 37218 . 25899 . 16614 . 093675 . 041740 . 010457	0. 35088 .30456 .25105 .19792 .14588 .098523 .058224 .027088 .0070666	0. 84132 . 66499 . 51053 . 37767 . 26566 . 17424 . 10259 . 049926 . 015504	0. 24927 .14688 .070298 .017218 015551 031136 032906 024645 011035		
				k=0.36						
0.1 .2 .3 .4 .5 .6 .7 .8	1, 0671 , 96403 , 86160 , 75972 , 65815 , 55637 , 45330 , 34634 , 22778	-0.11071 16298 20630 23993 20288 27363 26969 24642 19243	0.042228 .027934 .017956 .011013 .0063034 .0032579 .0014338 .00047434 .000080716	0. 21401 .16742 .12745 .093337 .064718 .041408 .023307 .010373 .0025984	0. 83394 . 65750 . 50339 . 37023 . 25752 . 16512 . 093079 . 041458 . 010388	0, 29112 , 25654 , 21455 , 16994 , 12607 , 085600 , 050808 , 023727 , 0062091	0.84153 .66627 .51262 .38017 .26844 .17685 .10474 .051359	0. 19224 .10323 .038111 0057839 031221 041114 038632 027377 011873		
		·	·	k=0.32	<u> </u>	<u> </u>	<del>'</del>	<del>!</del>		
0.1 .2 .3 .4 .5 .6 .7	1. 0845 . 98207 . 88011 . 77850 . 67690 . 57468 . 47057 . 36168 . 23952	-0.16335 22718 26250 28856 30430 30813 29747 20747 20622	0. 047477 . 032523 . 021755 . 013986 . 0034806 . 0047159 . 0022870 . 00086655 . 00018225	0. 19161 14957 11364 083076 057511 036740 020650 091777 0022962	0. 83258 -65655 -50139 -38845 -25611 -16430 -092482 -041178 -010314	0. 22710 . 20533 . 17477 . 14025 . 10510 . 071940 . 042991 . 020187 . 0053115	0. 84325 . 66906 . 51612 . 38401 . 27225 . 18028 . 10745 . 053123 . 016845	0. 13065 .056023 .0028801 031155 048051 045130 030513 012849		
				k=0.28				· · · · · · · · · · · · · · · · · · ·		
0.1 .2 .3 .4 .5 .6 .7 .8	1. 1078 1. 0058 . 90422 . 80265 . 70073 . 55769 . 49205 . 38056 . 25387	-0. 26523 30000 32667 34446 35225 34839 33015 29244 22272	0. 051744 .036348 .024978 .016540 .010372 .0059835 .0030403 .0012159 .00027249	0.16984 .13225 .10027 .073161 .050555 .032242 .018093 .0080289 .0020056	0. 83284 .65471 .50009 .36710 .25495 .16328 .091940 .040915 .010245	0.15751 .14985 .13180 .10828 .082571 .057313 .034632 .016414 .0043502	0. 84743 .67411 .52167 .38964 .27754 .18483 .11094 .055339 .017746	0. 062999 .00376233 036387 069668 065418 065151 052654 034189 014009		
				k=0.24				,		
0. 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8	1. 1396 1. 0380 . 93643 . 83456 . 73187 . 62742 . 51951 . 40444 . 27184	-0.359893847440187410444092839963309623228624300	0. 054843 . 039262 . 027507 . 018594 . 011918 . 0070337 . 0036728 . 0015114 . 00034985	0. 14865 . 11545 . 087322 . 063573 . 043844 . 027810 . 015634 . 0069261 . 0017273	0.83601 .65583 .50013 .36662 .25432 .16270 .091538 .040697 .010186	0, 080237 .083468 .084407 .073094 .057876 .041317 .025521 .012311 .0033103	0.85536 .68269 .53034 .30792 .28498 .19102 .11556 .058193 .018885	-0. 013014 055387 081216 092506 091405 080231 061597 038616 016424		
				k=0.20						
0.1 .2 .3 .4 .5 .6 .7 .8	1. 1847 1. 0830 . 98104 . 87824 . 77400 . 66724 . 55588 . 43576 . 29519	-0. 47320 48637 49316 49112 47956 45656 41908 36130 26886	0. 056532 . 041076 . 029205 . 029040 . 013047 . 0078484 . 0041572 . 0017422 . 00041096	0, 12795 .099080 .074748 .054292 .037358 .023781 .013266 .0058684 .0014604	0. 84412 . 66048 . 50264 . 36780 . 26474 . 16275 . 091452 . 040624 . 010154	-0.0082108 .018512 .030608 .033313 .030037 .023342 .015318 .0077280 .0021518	0.86976 .69696 .54396 .41036 .29575 .19971 .12187 .062012 .020376	-0. 10115 - 12455 - 13410 - 13162 - 11907 - 088580 - 072616 - 044140 - 017216		

 $<sup>{}^{2}</sup>$  Corrections to table II of NAOA TN 2590 have been incorporated herein.

TABLE II.—VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS\*—Continued

		. — —									
x <sub>1</sub> (wing chords)	k¹L4′	k <sup>2</sup> L4'	k2N1	ĿN₂	kºNi'	k2N1'	k²Nı'	₽N'			
	k=0.18										
0.1 .2 .3 .4 .5 .6 .7 .8	1. 2147 1. 1127 1. 0102 . 90658 . 80115 . 69268 . 57899 . 45551 . 30978	-0, 53882 -, 54727 -, 54743 -, 53940 -, 52183 -, 49284 -, 44919 -, 33485 -, 28480	0.056745 .041491 .029680 .020491 .013424 .0081229 .0043332 .0018283 .00043445	0.11774 .091031 .068581 .049747 .034188 .021694 .012115 .0063512 .0013308	0, 85115 .68501 .50541 .36942 .25563 .16318 .091621 .040672 .010160	-0.058372 -0.021025 .00029990 .010968 .014442 .013299 .0096244 .0051801 .0015101	0. 87830 .70742 .55362 .41893 .30300 .20544 .12596 .064450 .021314	0, 15168, 16446, 16454, 15463, 13540, 10961, 079234, 047492, 018316			
				k=0.16							
0.1 .2 .3 .4 .5 .6 .7 .8	1. 2518 1. 1494 1. 0460 94113 . 33405 . 72335 . 60667 . 47905 . 32712	-0. 61565 61627 60969 59494 57070 53494 48430 41244 30356	0. 056465 .041518 .029865 .020730 .013654 .0033118 .0044564 .0018911 .00045197	0. 10760 .083052 .062474 .045256 .031063 .019688 .019981 .0048451 .0012036	0.88128 .67174 .50980 .37217 .25723 .16404 .092024 .040814 .010189	-0.11399 064748 033187 013629 0025913 .0022897 .0033999 .0023970 .00081027	0. 89533 .72110 .56604 .42980 .31204 .21250 .13094 .067382 .022432	-0. 20814 20927 19952 18052 15403 12205 086886 051395 019606			
			,	k=0.14		<del></del> _		·			
0.1 .2 .3 .4 .5 .6 .7 .8	1. 2988 1. 1954 1. 0907 98410 87475 .76111 .64057 .50774 .34814	-0.70384 69678 68261 66030 62843 58483 52610 44543 32610	0, 055619 041107 029714 020725 013716 0083900 0045205 0019277 00046303	0. 097485 .075111 .056413 .040805 .027971 .017703 .0098625 .0043467 .0010787	0.87569 .68167 .51650 .37652 .25990 .16555 .092767 .041105 .010252	-0.17684 11401 070717 041207 021854 01000 0035341 00069611 .000033518	0. 91518 .73931 .58228 .44376 .32354 .22186 .13712 .070885	-0. 27248 26060 23947 21062 17574 13676 095915 056032 021160			
		·		k=0.12		<u> </u>		<u></u>			
0.1 .2 .3 .4 .5 .6 .7	1. 3599 1. 2548 1. 1481 1. 0390 92650 80889 68325 54349 37434	-0.80908 79327 77034 73925 69846 64572 57728 48600 35395	0.054111 .040180 .029172 .020435 .013583 .0083434 .0045147 .0019336 .0046633	0. 087342 . 067170 . 050365 . 036374 . 024898 . 015736 . 0087552 . 0038536 . 00095528	0. 89653 . 69636 . 52661 . 38324 . 26414 . 16802 . 094032 - 041617 . 010364	-0.24965 17090 11399 072883 043813 024051 011444 0042175 00085127	0. 94262 . 76408 . 60402 . 46224 . 33853 . 23279 . 14501 . 075541	-0.34769820922866824638201701644710686061694023050			
				<b>k</b> =0.10				-			
0.1 .2 .3 .4 .5 .6 .7 .8	1. 4421 1. 3345 1. 2246 1. 1118 99473 .87185 .73896 .59038 .40823	-0. 93930 91314 87981 83916 78657 72260 64222 53772 38961	0. 051808 .038635 .028163 .018906 .013214 .0081463 .0044256 .0019023 .00046075	0. 077093 .059172 .044289 .031034 .021874 .0076525 .0033639 .00083249	0. 92732 . 71843 . 54212 . 39376 . 27091 . 17204 . 096139 . 042488 . 010573	-0.33689 23884 16551 11048 069811 040637 020758 003561 0018817	0. 98172 . 79882 . 63413 . 48748 . 35879 . 24805 . 15644 . 081603 . 027709	-0. 43869 39432 34445 29046 23390 17866 12062 068886 025476			
				k=0.09							
0.1 .2 .3 .4 .5 .6 .7 .8	1. 4948 1. 3853 1. 2734 1. 1580 1. 0379 91101 . 77391 . 61956 . 42933	-1. 0178 98569 94624 89837 84038 76970 68214 56961 41167	0. 050302 .037586 .027450 .019339 .012926 .0078844 .0043442 .0018706 .00045375	0.071892 .055123 .041220 .029696 .020278 .012788 .0070997 .0031187 .00077151	0. 94835 .73385 .55291 .40116 .27572 .17494 .097670 .043128 .010721	-0. 38822 27870 19586 13245 084961 050287 026166 010754 0024828	1.0078 .82175 .65892 .50385 .37182 .25780 .16205 .085253 .029092	-0. 49263 43802 37902 31694 25334 18997 12901 073265 026971			
				k=0.08							
0.1 .2 .3 .4 .5 .6 .7 .8	1.5585 1.4467 1.3320 1.2133 1.0894 95808 .81555 .65421 .45434	-1, 1089 -1, 0699 -1, 0226 -, 98358 -, 99323 -, 82490 -, 72902 -, 60713 -, 43769	0. 048516 . 036323 . 026575 . 018755 . 012557 . 0077690 . 0042339 . 0018264 . 00044387	0.068611 .051020 .038115 .027435 .018718 .011795 .0055434 .0028721 .00070995	0. 97466 . 75290 . 56666 . 41064 . 28193 . 17869 . 099667 . 043974 . 010923	-0.44665 32399 22987 15731 10209 061178 032263 013450 0031549	1. 0400 .84998 .67789 .52376 .38758 .26952 .16996 .059728 .030732	-0. 55438 48821 41882 34755 27588 20558 13882 078426 028735			

<sup>&</sup>lt;sup>2</sup> Corrections to table II of NACA TN 2590 have been incorporated herein.

# TABLE II.—VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS2—Continued

x <sub>1</sub> (wing chords)	$k^2L_{\bf i}'$	k2 L4'	k³N₁	k2N2	k2N2'	k2N4'	k2N\$'	k2N6′		
	k=0.07									
0.1 .2 .3 .4 .5 .6 .7 .8	1. 6372 1. 5222 1. 4039 1. 2812 1. 1525 1. 0155 . 86612 . 69629 . 48460	-1. 2166 -1. 1697 -1. 1154 -1. 0522 - 97833 - 89087 78518 - 65224 46902	0. 046407 .034808 .025510 .018083 .012093 .0074936 .0040903 .0017668 .00043011	0.061221 .046838 .034957 .025138 .017137 .010789 .0058605 .0026232 .00064778	1. 0085 . 77768 . 58447 . 42300 . 29005 . 18363 . 10232 . 045097 . 011194	-0. 51450 37651 26945 18604 12185 073725 039270 016551 0039242	1, 0808 . 88543 . 70800 . 54851 . 40706 . 28394 . 17964 . 095173 . 032722	-0. 62653 54704 46584 38366 30257 22413 15051 084589 030851		
				k=0.08						
0.1 .2 .3 .4 .5 .6 .7 .8	1. 7369 1. 6178 1. 4947 1. 3665 1. 2316 1. 0874 92934 74889 52222	-1. 3476 -1. 2914 -1. 2276 -1. 1546 -1. 0704 97200 85442 70790 50782	0. 043916 .032999 .024225 .017151 .011519 .0071485 .0039074 .0016905 .00041195	0. 055674 . 042548 . 031722 . 022791 . 015522 . 0097646 . 0054083 . 0023700 . 00058482	1. 0528 .81043 .60811 .43945 .30095 .19029 .10591 .046631 .011560	-0, 59555 -, 43902 -, 31647 -, 22011 -, 14523 -, 088553 -, 047545 -, 020200 -, 0048319	1, 1337 .93114 .74657 .58003 .43178 .30216 .19182 .10199 .035201	-0.71312 61787 52218 42743 33504 24678 16484 092200 033467		
				k=0.05						
0.1 .2 .3 .4 .5 .6 .7 .8	1. 8681 1. 7481 1. 6135 1. 4779 1. 3347 1. 1808 1. 0113 . 81660 . 57076	-1. 5127 -1. 4452 -1. 3697 -1. 2845 -1. 1875 -1. 0754 94288 77923 55760	0.040965 .030833 .022671 .016076 .010812 .0067195 .0036780 .0015940 .00038898	0. 049908 .038098 .028375 .020365 .013857 .0087093 .0048198 .0021103 .00052030	1. 1131 ( . 85516 . 64058	-0. 69565 51610 37435 26195 17391 10670 057655 024646 0059388	1. 2047 . 99223 . 79785 . 62178 . 46433 . 32603 . 20770 . 11084 . 038400	-0.82080 70625 58303 48245 37600 27545 18307 10190 036823		
				k=0.045		·				
0.1 .2 .3 .4 .5 .6 .7 .8	1, 9509 1, 8220 1, 6381 1, 5478 1, 3992 1, 2391 1, 0624 85866 60094	-1. 6137 -1. 5394 -1. 4563 -1. 3643 -1. 2596 -1. 1392 99749 82334 58847	0. 089279 .029589 .021773 .015450 .010398 .0064672 .0035423 .0015355 .00037489	0.046915 .035790 .026641 .019112 .012999 .0031656 .0045163 .0019771 .00048738	1. 1519 .88412 .66167 .48308 .32592 .20566 .11423 .050194 .012420	-0.75597 56248 40911 25704 19108 11754 063634 027313 0065977	1. 2502 1. 0312 . 83043 . 64818 . 48487 . 34101 . 21765 . 11636 . 040395	-0. 88599 75988 63613 51605 40107 29306 19428 10789 038898		
		·		<b>k</b> =0.04		·				
0.1 .2 .3 .4 .5 .6 .7 .8	2, 0498 1, 9162 1, 7771 1, 6310 1, 4759 1, 3085 1, 1230 90867 63667	-1. 7318 -1. 6498 -1. 5590 -1. 4580 -1. 3443 -1. 2143 -1. 0619 87538 62490	0.037426 .028214 .020776 .014754 .0099366 .0061837 .003896 .0014699 .00035870	0.043824 .033413 .024863 .017824 .012116 .0076080 .0042053 .0018402 .00045342	1. 1991 . 91933 . 68736 . 49510 . 33802 . 21310 . 11829 . 051938 . 012837	-0.8284 61611 44926 31602 21088 13005 070624 030368 0073584	1. 3051 1. 0781 86957 67986 50939 35891 22949 12283 042760	-0.96172 82232 68640 55525 43040 31370 20746 11493 041344		
				k=0,035						
0.1 .2 .3 .4 .5 .6 .7 .8 .9	2. 1705 2. 0309 1. 8854 1. 7323 1. 5891 1. 3926 1. 1965 96920 67984	-1. 8730 -1. 7818 -1. 6814 -1. 5703 -1. 4460 -1. 3045 -1. 1393 93804 66883	0. 035377 .025693 .019670 .013978 .0094210 .0058673 .0032180 .0013972 .00034181	0.040614 .030946 .023010 .016490 .011203 .0070313 .0038855 .0016886 .00041808	1. 2576 .96307 .71933 .51765 .35309 .22248 .12336 .054136 .013371	-0. 90842 67943 49662 35013 22417 14473 078792 033945 0082444	1. 3728 1. 1353 . 91759 . 71859 . 53936 . 38069 . 24387 . 13089 . 045619	-1. 0516 89653 74023 60203 46546 33841 22226 12341 044291		
				k=0.03						
0, 1 .2 .3 .4 .5 .6 .7 .8	2 3217 2 1748 2 0210 1. 8537 1. 6354 1. 4974 1. 2880 1. 0445 . 73346	-2 0464 -1. 9440 -1. 8321 -1. 7088 -1. 5714 -1. 4158 -1. 2350 -1. 0157 72327	0. 033095 .024089 .018428 .013105 .0083380 .0055076 .0030229 .0013127 .00032172	0. 037250 .028365 .021080 .015088 .010253 .0064317 .0035223 .0015530 .00038174	1. 3320 1. 0188 . 76018 . 54651 . 37243 . 23442 . 12991 . 056942 . 014072	-1. 0087 75625 56399 39141 26225 16248 088626 038279 0092997	1. 4584 1. 2086 97812 76729 57694 40796 26183 14080 049169	-1. 1611 98712 81945 65939 50852 36881 24274 13388 047946		

<sup>&</sup>lt;sup>2</sup> Corrections to table II of NAOA TN 2590 have been incorporated herein.

TABLE II.—VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS Continued

(wing chords)	kıI4'	₽L/	k2N1	₽N₂	k2N2'	₽N¦	k²Ni′	k2N4'			
1	k=0.025										
0.1 .2 .3 .4 .5 .6 .7 .8	2. 5186 2. 3615 2. 1968 2. 0226 1. 8360 1. 6329 1. 4061 1. 1415 . 80256	-2 2672 -2 1509 -2 0244 -1. 8858 -1. 7319 -1. 5584 -1. 3578 -1. 1152 79325	0. 030530 .023067 .017023 .012113 .0081744 .0050971 .0027995 .0012168 .00029798	0. 033691 .025634 .019039 .013629 .0092506 .0058001 .0032017 .0013387 .00034401	1. 4302 1. 0926 .81427 .58480 .39814 .25036 .13863 .060719 .014969	-1. 1351 85288 62606 44322 29765 18471 10094 043673 010652	1. 5709 1. 3041 1. 0571 . 83081 . 62581 . 44336 . 28509 . 15361 . 053757	-1. 2095 -1. 1018 91231 78225 56335 40761 26766 14728 052627			
				k=0.024							
0.1 .2 .3 .4 .5 .6 .7 .8	2. 5654 2. 4060 2. 2386 2. 0614 1. 8717 1. 6650 1. 4341 1. 1646 81890	-2, 3191 -2, 1996 -2, 0697 -1, 9274 -1, 7698 -1, 5921 -1, 3867 -1, 1388 -, 80980	0. 029968 . 022650 . 016717 . 011897 . 0080800 . 0050076 . 0027499 . 0011959 . 00029268	0.032942 .025085 .018614 .013323 .0090426 .0056691 .0031298 .0013666 .00033638	1. 4538 1. 1103 . 81026 . 59403 . 40435 . 25422 . 14072 . 061661 . 015205	-1.1646 87540 65969 45629 30586 18987 10382 044896 010948	1. 6979 1. 3263 1. 0760 84591 . 63746 . 45176 . 29060 . 15664 . 054835	-1, 3319 -1, 1228 -, 93410 -, 74943 -, 57629 -, 41676 -, 27355 -, 15046 -, 053743			
				k=0.023		·					
0.1 .2 .3 .4 .5 .6 .7 .8	2. 6154 2. 4533 2. 2832 2. 1030 1. 9098 1. 6993 1. 4640 1. 1890 . 83630	-2.3744 -2.2013 -2.1179 -1.9717 -1.8100 -1.6279 -1.4176 -1.1638 82741	0, 029397 . 022222 . 016403 . 011675 . 0078810 . 0049150 . 0026999 . 0011738 . 00028746	0. 032188 .024488 .018183 .013014 .0088317 .0055365 .0030559 .0013347 .00032830	1. 4789 1. 1293 84122 60391 41099 25834 14299 062634 015457	-1. 1959 89930 66067 46808 31458 19536 10683 046232 011263	1. 6266 1. 3512 1. 0961 . 86206 . 64988 . 46075 . 29650 . 15988 . 055995	-1. 3663 -1. 1573 95728 76768 58999 42646 27979 15382 054921			
				k=0.022		··	· ···· · · · · · · · · · · · · · · · ·				
0.1 .2 .3 .4 .5 .6 .7 .8	2. 6689 2. 5040 2. 3308 2. 1474 1. 9506 1. 7360 1. 4959 1. 2162 . 85489	-2. 4331 -2. 3065 -2. 1691 -2. 0190 -1. 8529 -1. 6661 -1. 4505 -1. 1905 84618	0. 028810 .021781 .016080 .011447 .0077275 .0048202 .0026477 .0011512 .00028163	0.031420 .023901 .017745 .012699 .0086171 .0054014 .0029813 .0013020 .00032059	1. 5060 1.1496 .85620 .61449 .41810 -26275 .14539 .063681 .015677	-1. 2292 92473 67963 48169 32384 20118 11008 047664 011643	1. 6575 • 1. 3774 1. 1177 87938 66318 47035 30280 16335 057233	-1. 4028 -1. 1877 98189 78694 60456 43681 28645 15741 056173			
				k=0.021							
0.1 .2 .3 .4 .5 .6 .7 .8	2. 7263 2. 5584 2. 3820 2. 1949 1. 9942 1. 7762 1. 5300 1. 2482 . 87481	-2, 4959 -2, 3653 -2, 2240 -2, 0695 -1, 8888 -1, 7069 -1, 4856 -1, 2191 -, 86630	0.028205 .021326 .015747 .011211 .0075893 .0047218 .0025940 .0011283 .00027604	0. 030639 .023305 .017300 .012379 .0083984 .0052642 .0029055 .0012685 .00031230	1. 5352 1. 1715 . 87230 . 62591 . 42579 . 26754 . 14801 . 064818 . 015974	-1. 2645 95181 69978 49617 33369 20737 11350 049141 011994	1. 6906 1. 4054 1. 1409 . 89792 . 67742 . 48085 . 30955 . 16705 . 058556	-1. 4419 -1. 2201 -1. 0082 80765 62018 44788 29356 16125 057515			
				k=0.020							
0.1 .2 .3 .4 .5 .6 .7 .8	2. 7880 2. 6169 2. 4370 2. 2461 2. 0411 1. 8174 1. 5667 1. 2733 : 89624	-2.5632 -2.4285 -2.2628 -2.1237 -1.9480 -1.7508 -1.5234 -1.2498 88788	0.027582 .020859 .015404 .010968 .0074064 .0046212 .0025391 .0011040 .00027078	0.029843 .022596 .016846 .012053 .0081758 .0051240 .003277 .0012350 .00030338	1. 5665 1. 1951 . 88968 . 63824 . 43408* . 27269 . 15082 . 065884 . 016262	-1. 3025 96076 72136 51164 34422 21399 11717 050796 012401	1. 7264 1. 4357 1. 1659 . 91792 . 69276 . 49172 . 31681 . 17104 . 059980	-1. 4836 -1. 2547 -1. 0363 82980 63658 45972 30120 16537 058960			
			·	k=0.019							
0.1 .2 .3 .4 .5 .6 .7	2. 8548 2. 6801 2. 4964 2. 3013 2. 0918 1. 8629 1. 6063 1. 3058 . 91929	-2 6357 -2 4965 -2 3461 -2 1820 -2 0011 -1 7980 -1 5641 -1 2828 - 91116	0. 026938 . 020375 . 015049 . 010717 . 0072381 . 0045165 . 0024821 . 0010798 . 00026495	0. 029032 .022076 .016384 .011721 .0079503 .0049818 .0037487 .0012001 .00029461	1. 6005 1. 2208 . 90853 . 65164 . 44309 . 27829 . 15390 . 067334 . 016595	-1. 3432 -1. 0118 74445 52822 35549 22108 12108 052500 012818	1,7851 1,4684 1,1929 93954 ,70933 ,50367 ,32464 ,17533 ,081614	-1. 5285 -1. 2921 -1. 0666 83362 65485 47251 30943 16981 060518			

<sup>&</sup>lt;sup>2</sup> Corrections to table II of NACA TN 2590 have been incorporated herein.

TABLE II.—VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS -Continued

x <sub>1</sub> (wing chords)	£2L4′	<i>№L.</i> ′	$k^2N_1$	k2N2	k³NY	k°N₁′	kºNg'	₽N6		
· k=0.018										
0.1 .2 .3 .4 .5 .6 .7 .8	2. 9271 2. 7486 2. 5507 2. 3612 2. 1467 1. 9122 1. 6492 1. 3410 . 94427	-2.7139 -2.5699 -2.4144 -2.2451 -2.0583 -1.6081 -1.318693636	0. 026272 .019874 .014681 .010468 .0070622 .0044074 .0024221 .0010533 .00025855	0. 028204 .021444 .015914 .011383 .0077206 .0048376 .0026691 .0011654 .00028609	1. 6375 1. 2488 . 92904 . 66621 4.5289 . 28440 . 15725 . 068795 . 016983	-1. 3871 -1. 0452 76934 54607 36764 22870 12529 054332 013238	1.8071 1.5038 1.2209 .96296 .72728 .51665 .33310 .17999 .063170	-1. 5768 -1. 3323 -1. 0993 87937 67428 48629 31833 17462 062205		
				k=0.017			,			
0.1 .2 .3 .4 .2 .6 .7 .8	3. 0059 2. 8222 2. 6308 2. 4263 2. 4263 1. 6958 1. 6958 1. 3792 . 97142	-2 7887 -2 6496 -2 4887 -2 3135 -2 1206 -1 9044 -1 6559 -1 3575 - 96373	0. 025583 .019366 .014300 .010187 .0068811 .0042948 .0023608 .0010270 .00025201	0. 027357 .020797 .015431 .011037 .0074851 .0048893 .0025869 .0011292 .00027727	1. 6779 1. 2791 . 95150 . 68216 . 46364 . 29111 . 16094 . 070418	-1. 4345 -1. 0814 79625 56537 38073 23691 12982 056294 013714	1, 8530 1, 5426 1, 2541 , 98850 , 74683 , 53072 , 34232 , 18504 , 064967	-1. 6292 -1. 3759 -1. 1347 90732 69539 50130 32802 17985 064042		
				k=0.016						
0.1 .2 .3 .4 .5 .6 .7 .8	3. 0922 2. 9048 2. 7075 2. 4976 2. 2717 2. 0244 1. 7468 1. 4210 1. 0011	-2.8913 -2.7384 -2.5695 -2.3881 -2.1884 -1.9649 -1.7081 -1.3999 99364	0. 024870 018820 013906 0099067 0066931 0041779 0022970 00099935 00024528	0. 026488 .020134 .014938 .010683 .0072438 .0045379 .0025028 .0010925 .00026819	1. 7222 1. 3125 . 97610 . 69962 . 47542 . 29842 . 16495 . 072133 . 017780	-1. 4861 -1. 1208 82552 58637 39501 24659 13479 058488 014281	1. 9033 1. 5850 1. 2891 1. 0165 - 76831 - 54620 - 35246 - 19059 - 066939	-1. 6862 -1. 4233 -1. 1733 93768 71834 51761 33851 18553 066043		
<del></del>		·		k=0.015		<del></del>		·		
0.1 .2 .3 .4 .5 .6 .7 .8	3. 1871 2. 9948 2. 7918 2. 5760 2. 3436 2. 0889 1. 8028 1. 4669 1. 0337	-2 9925 -2 8316 -2 6884 -2 4701 -2 2028 -2 0312 -1 7884 -1 4485 -1 0285	0. 024129 018262 013495 0096154 0064969 0040581 0022300 00097034 00023810	0. 025598 .019455 .014432 .010320 .0069973 .0043828 .0024172 .0010550 .00025902	1. 7712 1. 3494 1. 0033 . 71894 . 48843 . 30652 . 16937 . 074050 . 018238	-1. 5426 -1. 1638 85762 60928 41063 25689 14022 060865 014878	1. 9588 1. 6319 1. 3277 1. 0473 . 79191 . 56320 . 36358 . 19668 . 069109	-1. 7485 -1. 4752 -1. 2155 97099 74351 53548 35006 19177 068234		
<u>-</u>				k=0.014				·		
0.1 .2 .3 .4 .5 .6 .7 .8	3. 2924 3. 0943 2. 8863 2. 6629 2. 4230 2. 1603 1. 8648 1. 5177 1. 0697	-3. 1042 -2. 9387 -2. 7563 -2. 5603 -2. 3451 -2. 1045 -1. 8288 -1. 4980 -1. 0627	0. 023357 .017880 .013068 .0033118 .0062926 .0039288 .0021603 .00094017 .00023065	0. 024682 .018757 .013912 .0099470 .0067438 .0042238 .0023293 .0010165 .00024963	1. 8255 1. 3905 1. 0335 . 74045 . 50292 . 31554 . 17433 . 076211 . 018772	-1. 6047 -1. 2111 89270 63449 42775 26642 14615 063451 015509	2. 0203 1. 6838 1. 3705 1. 0814 . 81805 . 68200 . 37587 . 20341 . 071516	-1. 8172 -1. 5324 -1. 2621 -1. 0077 77128 55525 36282 19869 070646		
				<b>k</b> =0.013	,					
0.1 .2 .3 .4 .5 .6 .7 .8	3. 4097 3. 2053 2. 9894 2. 7594 2. 5116 2. 2398 1. 9339 1. 5743 1. 1098	-3. 2286 -3. 0533 -2. 8657 -2. 6607 -2. 43365 -2. 1860 -1. 8991 -1. 5553 -1. 1031	0. 022555 .017074 .012621 .0089943 .0060789 .0037957 .0020873 .00090843 .00022294	0. 023739 .018037 .013378 .0095637 .0064833 .0040601 .0022387 .00097697 .00023984	1. 8864 1. 4364 1. 0874 76464 51920 32570 17993 .078678 .019430	-1. 6735 -1. 2935 93165 66238 44665 27828 15267 066270 016150	2 0891 1. 7417 1. 4181 1. 1195 .84715 .60292 .38955 . 21090 .074147	-1. 8934 -1. 9960 -1. 3138 -1. 0486 80219 57725 37704 20638 073381		
				k=0.012						
0.1 .2 .3 .4 .5 .6 .7 .8	3. 5417 3. 3301 3. 1065 2. 8682 2. 6112 2. 3291 2. 0114 1. 6379 1. 1549	-3. 3676 -3. 1841 -2. 9871 -2. 7734 -2. 5390 -2. 2775 -1. 9780 -1. 6196 -1. 1484	0. 021712 .016489 .012153 .0086626 .0058552 .0036586 .0020110 .00687519 .00021465	0. 022764 . 017294 . 012825 . 0091670 . 0062137 . 0038909 . 0021455 . 00033623 . 00022000	1. 9549 1. 4882 1. 1056 . 79170 . 53749 . 33710 . 18618 . 081361 . 020058	-1.7505 -1.3221 97518 69356 46784 29157 16003 069513 016976	2.1667 1.8071 1.4720 1.1624 .87996 .62656 .40496 .21933 .077175	-1. 9786 -1. 6669 -1. 3716 -1. 0942 83673 60182 39290 21488 076378		

<sup>&</sup>lt;sup>2</sup> Corrections to table II of NACA TN 2590 have been incorporated herein.

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TABLE II.—VALUES OF FUNCTIONS FOR AILERON FLUTTER CALCULATIONS Concluded

(wing chords)	. PL	k2L4'	₽N <sub>1</sub>	k²N₂	₽N¦	βNζ	₽N{	PN4
				k=0.011				
0.1 .2 .3 .4 .5 .6 .7 .8	3. 6917 3. 4719 3. 2395 2. 9917 2. 7242 2. 4304 2. 0995 1. 7099 1. 2059	-3. 5250 -3. 3322; -3. 1252 -2. 9010 -2. 6551 -2. 3810 -2. 0674 -1. 6923 -1. 1998	0. 020830 .015774 .011663 .0083139 .0056202 .0035105 .0019308 .00084042 .00020593	0. 021752 .016523 .012251 .0087665 .0059347 .00371.65 .0020485 .0020485 .00021976	2.0329 1.5471 1.1491 .82262 .55834 .35009 .19327 .084414 .020733	-1.8373 -1.3881 -1.0243 72876 49176 30658 16835 077263 017948	2, 2549 1, 8814 1, 6331 1, 2112 91728 65338 42248 23988 , 080575	-2.0748 -1.7472 -1.4370 -1.1468 87576 62964 41087 23588 079806
				<b>k</b> =0.010			,	
0.1 .2 .3 .4 .5 .6 .7 .8	3. 8640 3. 6347 3. 3922 3. 1334 2. 8538 2. 5467 2. 2004 1. 7924 1. 2645	-3. 7050 -3. 5015 -3. 2832 -3. 0469 -2. 7880 -2. 4996 -2. 1698 -1. 7758 -1. 2586	0. 019900 .018072 .011145 .0079462 .0053724 .0033557 .0018459 .00080360 .00019691	0.020700 015722 011656 0083296 0056448 0035340 0019482 00084993 00020898	2, 1229 1, 6152 1, 1993 , 85836 , 88247 , 36512 , 20154 , 088031 , 021638	-1. 9363 -1. 4636 -1. 0802 76880 51893 32364 17775 077263 018934	2, 3564 1, 9668 1, 6034 1, 2672 96003 684415 44253 , 23988 , 084451	-2. 1847 -1. 8390 -1. 5117 -1. 2048 92053 66149 43150 23688 083757

 $<sup>^2</sup>$  Corrections to table II of NACA TN 2590 have been incorporated herein.